Decision problems and some mathematical games

Real life problems

Mateusz Topolewski

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Matching pennies

Each player has a penny and must secretly turn the penny to heads or tails. The players then reveal their choices simultaneously. If the pennies match (both heads or both tails), then Player 1 keeps both pennies. If the pennies do not match (one heads and one tails) Player 2 keeps both pennies.

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Strategy	H	T
H	$(1,-1)$	$(-1,1)$
T	$(-1,1)$	$(1,-1)$

Chicken

Two drivers drive towards each other on a collision course: one must swerve, or both may die in the crash, but if one driver swerves and the other does not, the one who swerved will be called a 'chicken', meaning a coward.

	Swerve	Straight
Swerve	(0,0)	(-1,1)
Straight	(1,-1)	(-10,-10)

Volunteer's Dilemma

The volunteer's dilemma game models a situation in which each of N players faces the decision of either making a small sacrifice from which all will benefit, or freeriding.

	at least one other person cooperates	all others defect
Cooperate	0	0
Defect	1	-10

Volunteer's Dilemma — special cases

$N = 2$

	Cooperate	Defect
Cooperate	(0,0)	(0,1)
Defect	(1,0)	(-10,-10)

$N = 3$

Third cooperates

	Cooperate	Defect
Cooperate	(0,0,0)	(0,1,0)
Defect	(1,0,0)	(1,1,0)

Third defect

	Cooperate	Defect
Cooperate	(0,0,1)	(0,1,1)
Defect	(1,0,1)	(-10,-10,-10)

Prisoner's Dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge. They hope to get both sentenced to a year in prison on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to: betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent.

	Stay Silent	Betray
Stay Silent	(-1,-1)	(-5,0)
Betray	(0,-5)	(-3,-3)

Iterated Prisoner's Dilemma

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Question

Is there anything better?